Wave Structures for Nonlinear Schrodinger Types Fractional Partial Differential Equations Arise in Physical Sciences

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ABSTRACT

Nonlinear partial differential equations are mostly renowned for depicting underlying behavior of nonlinear phenomena relating to the nature of real world. In this paper, we discuss for analytic solutions of fractional order nonlinear Schrodinger types equations such as the space-time fractional nonlinear Schrodinger equation and the (2+1)-dimensional time-fractional Schrodinger equation. The considered equations are converted into ordinary differential equations with the help of wave variable transformation and then the recently established rational $(D_\xi^\alpha G/G^2)$-expansion method is employed to construct the exact solutions. The obtained solutions are appeared in the forms of trigonometric function, hyperbolic function and rational function which are compared with those of literature and claimed to be different. The graphical representations of the solutions are finally brought out for their physical appearances. The applied method is seemed to be efficient, concise and productive which might be used for further research.

INTRODUCTION

In many fields of science, it is important to integrate and solve nonlinear partial derivative differential equations that contain derivative by time. The results of fractional nonlinear wave models have presented an advanced success in applied mathematics and engineering, mathematical physics, soliton physics. These fractional nonlinear wave models become much devotion in several other areas like signal processing, psychology, finance, acoustics,

biology systems, medical processes, chemical physics, biology, fluid mechanics, medical process, and many more (Laskin, 2002; Eslami et al., 2004). Recently, the search for a different type of solution of the fractional nonlinear Schrodinger's model has represented numerous scientists and researchers (Saxena & Kalla, 2010; Abdel-Salam et al., 2016; Younis et al., 2017; Rizvi et al., 2017). The established methods available in the literature are G'/G-expansion method for nonlinear fractional differential equations by Bekir and Guner (2013), the meshless method of lines (Mohyud-Din, 2012), Adomian decomposition scheme (Guo, 2019), the new generalized G'/G-expansion method (Alam et al., 2014; Alam, 2015; Alam & Li, 2019), the Darcy law method (Sheikholeslami, 2017), the reproducing kernel algorithm (Omar, 2019a), topological solitons for certain differential equation by Biswas et al. (2013a), Biswas et al. (2013b), Laplace-Adomian decomposition method (Shah et al., 2019), reproducing kernel Hilbert space method (Omar, 2019b), homotopy perturbation method (Golmankhaneh & Baleanu, 2011), improved sub-equation method (Karaagac, 2019), Schrodinger's equation (Rizvi et al., 2017; Li et al., 2019), generalized exponential rational function method (Ghanbari et al., 2019), the Kudryashov methods (Saha, 2016; Kudryashov, 2012), integral transform based decomposition methods are used for Schrodinger and other differential equations by Nuruddeen (2017); Nuruddeen & Nass (2017), Various phenomena such as shallow water waves and multicellular biology dynamics arising in the nonlinear physical science (Lu et al., 2017; Bazyar & Song, 2017), the $(G'/G, 1/G)$-expansion method (Zayed & Abdelaziz, 2012; Zayed et al., 2018; Zayed & Alurrfi, 2016), the Jacobi collocation method (Doha et al., 2014; Bhrawy et al., 2014a; Bhrawy et al., 2014b)], the improved tanh method (Islam & Akter, 2021a), the rational (G'/G)-expansion method (Jannah et al., 2021) etc. In lower the summary of this paper shown. In the 2nd section, conformable fractional derivative and methodology are displayed, in the 3rd section application of this method, in the 4th section graphical representations are shown, the 5th section is for conclusion.

PRELIMINARIES AND METHODOLOGY

Conformable fractional derivative

Let $\omega: [0, \infty) \to \boldsymbol{R}$ be a function. The $\alpha's$ order conformable derivative of u is defined by Khalil et al. (2014)

$$D_x^\alpha(\omega(x)) = \lim_{h \to 0} \frac{\omega(x + hx^{1-\alpha}) - \omega(x)}{h}$$

for all $x > 0$ and $\alpha \epsilon (0,1)$. Further, the following theorems gives some properties of conformable derivative:

Theorem 1. Let $\alpha \epsilon (0,1)$ and suppose $\omega(x)$ and $\varrho(x)$ are α-differentiable at $x > 0$. Then

(i). $D_x^\alpha(x^\varsigma) = \varsigma x^{\varsigma - \alpha}$, for all $\varsigma \epsilon \boldsymbol{R}$.

(ii). $D_x^\alpha(\beta) = 0, \beta$ for all constant function $\omega(x) = \beta$.

(iii). $D_x^\alpha(\beta \omega(x)) = \beta D_x^\alpha \omega(x)$, for all β constant.

(iv). $D_x^\alpha(\beta \omega(x) + \gamma \varrho(x)) = \beta D_x^\alpha \omega(x) + \gamma D_x^\alpha \varrho(x)$, for all $\beta, \gamma \epsilon R$.

(v). $D_x^\alpha(\varrho(x)\omega(x)) = \varrho(x) D_x^\alpha(\omega(x)) + \omega(x) D_x^\alpha(\varrho(x))$.

(vi). $D_x^\alpha\left(\frac{\omega(x)}{\varrho(x)}\right) = \frac{\varrho(x) D_x^\alpha \omega(x) - \omega(x) D_x^\alpha \varrho(x)}{\varrho^2(x)}, \varrho \neq 0$.

(vii). If, in addition to $\omega(x)$ differentiable, then $D_x^\alpha \omega(x) = x^{1-\alpha} \frac{d\omega}{dx}$.

Theorem 2. Let $\alpha \epsilon (0,1)$ such $\omega(x)$ is differentiable and also α-differentiable. Let $\varrho(x)$ be a function defined in the range of $\omega(x)$ also differentiable, then

$$D_x^\alpha(\omega(x).\varrho(x)) = x^{1-\alpha}\varrho'(x)\omega'(\varrho(x)).$$

Methodology

In this section, we introduce the newly established rational $(D_\xi^\alpha G/G^2)$-expansion method (Islam & Akter, 2021b) for finding exact analytic solutions of nonlinear partial differential equations. Consider the FNLEE in the independent variables $t, x_1, x_2, \dots, x_n$ as

$$Q(u_1, \dots, u_k, D_t^\alpha u_1, \dots, D_t^\alpha u_k, D_{x_1}^\beta u_1, \dots, D_{x_1}^\beta u_k, \dots D_{x_n}^\beta u_1, \dots D_{x_n}^\beta u_k, \dots) = 0, \qquad (2.2.1)$$

where $u_i = u_i(t, x_1, x_2, \dots, x_n), i = 1, \dots, k$ are unknown functions, F is a polynomial in u_i and it's various partial derivatives of fractional order.

Making use of the fractional composite transformation

$$u_i = u_i(t, x_1, x_2, \dots, x_n) = U_i(\xi), \xi = \xi(t, x_1, x_2, \dots, x_n) \qquad (2.2.2)$$

Eq. (2.2.1) is turned into the following ordinary differential equation of fractional order with respect to the variable ξ:

$$Q(u_1, \dots, u_k, D_\xi^\alpha u_1, \dots, D_\xi^\alpha u_k, D_\xi^\beta u_1, \dots, D_\xi^\beta u_k, \dots) = 0 \qquad (2.2.3)$$

We may, if possible, take the anti-derivative of Eq. (2.2.3) term by term one or more times and integral constant can be set to zero as soliton solutions are sought.

Now, we discuss the main steps of the above-mentioned method to investigate exact analytic solutions of FNLEEs as follows:

First step: Suppose the solution to be in the form

$$U(\xi) = \frac{\sum_{i=0}^n \iota_i (D_\xi^\alpha G/G^2)^i}{\sum_{i=0}^n \tau_i (D_\xi^\alpha G/G^2)^i}, \qquad (2.2.4)$$

where at least one of ι_n and τ_n is nonzero; $\iota_i(i = 0, 1, 2, \dots, n)$, $\tau_i(i = 0, 1, 2, \dots, n)$ and d are unknown parameters, and $G = G(\xi)$ satisfies the following auxiliary nonlinear ordinary differential equation of fractional order:

$$G^2 D_\xi^{2\alpha} G - (2G + \lambda)(D_\xi^\alpha G)^2 - \mu G^4 = 0 \qquad (2.2.5)$$

It can be written as

$$D_\xi^\alpha(D_\xi^\alpha G/G^2) = \mu + \lambda(D_\xi^\alpha G/G^2)^2 \qquad (2.2.6)$$

The nonlinear fractional composite wave variable transformation $G(\xi) = H(\eta)$, $\eta = \xi^\alpha/\alpha$ reduces Eq. (2.2.5) into the following ODE:

$$H^2 H'' - (2G + \lambda)H'^2 - \mu H^4 = 0 \qquad (2.2.7)$$

whose solutions are well-known. Since $D_\xi^\alpha G(\xi) = D_\xi^\alpha H(\eta) = H'(\eta)D_\xi^\alpha \eta = H'(\eta)$, with the aid of the solutions of Eq. (2.2.7), we can obtain the solutions of Eq. (2.2.5) as follows:

$$(D_\xi^\alpha G/G^2) = \sqrt{\mu/\lambda} \times \frac{A\cos\sqrt{\lambda\mu}\xi^\alpha/\alpha + B\sin\sqrt{\lambda\mu}\xi^\alpha/\alpha}{B\cos\sqrt{\lambda\mu}\xi^\alpha/\alpha - A\sin\sqrt{\lambda\mu}\xi^\alpha/\alpha}, \qquad \lambda\mu > 0 \qquad (2.2.8)$$

$$(D_\xi^\alpha G/G^2) = -\sqrt{|\lambda\mu|/\lambda} \times \frac{A\sinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + A\cosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + B}{A\sinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + A\cosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) - B}, \qquad \lambda\mu < 0 \qquad (2.2.9)$$

$$(D_\xi^\alpha G/G^2) = -\frac{A\alpha}{\lambda(A\xi^\alpha + B\alpha)}, \qquad\qquad \mu = 0, \lambda \neq 0 \quad (2.2.10)$$

Second step: Due to the theme of homogenous balance, Eq. (2.2.3) serves the value of n appeared in Eq. (2.2.4).

Third step: Use Eqs. (2.2.4) and (2.2.5) in Eq. (2.2.3) with the value of n found in second step to obtain a polynomial in $(D_\xi^\alpha G/G^2)$. Set each coefficient of this polynomial to zero and solve them by computer software Maple to gain the values of the unknown parameters available in Eq. (2.2.4).

Fourth step: Utilizing the values calculated at third step in Eq. (2.2.4) along with Eqs. (2.2.8)-(2.2.10) makes available exact analytic solutions to Eq. (2.2.1).

APPLICATION OF THE METHOD

The well-known space and time fractional nonlinear Schrodinger equation is

$$iD_t^\alpha u + D_x^{2\beta} u + 2|u|^2 u = 0, \, t > 0, 0 < \alpha, \beta \leq 1, \qquad\qquad (3.1.1)$$

where x is the spatial variable and t represents time. This equation occurs in non-linear optics, superconductivity and plasma physics. We introduce the fractional composite transformation as follows:

$$u(x,t) = e^{i\varphi} u(\xi), \, \varphi = m^{1/\beta} x + k^{1/\alpha} t, \, \xi = x + r^{1/\alpha} t, \qquad\qquad (3.1.2)$$

where r, k and m are arbitrary constants to be determined later. Eq. (3.1.1) with the aid of Eq. (3.1.2) reduces to one whose real part and imaginary part are given as:

$$-(k + m^2)u + D_\xi^{2\alpha} u + 2u^3 = 0, \qquad\qquad (3.1.3)$$

$$rD_\xi^\alpha u + 2mD_\xi^\alpha u = 0, \qquad\qquad (3.1.4)$$

where u and its various derivatives are functions of ξ. Eq. (3.1.4) yields $= -\frac{r}{2}$, so that Eq. (3.1.3) becomes

$$-(4k + r^2)u + 8u^3 + 4D_\xi^{2\alpha} u = 0. \qquad\qquad (3.1.5)$$

Balancing the terms $D_\xi^{2\alpha} u$ and u^3 provides $n = 1$ and the solution Eq. (2.2.4) takes the form

$$u(\xi) = \frac{\iota_0 + \iota_1(D_\xi^\alpha G(\xi)/G^2(\xi))}{\tau_0 + \tau_1(D_\xi^\alpha G(\xi)/G^2(\xi))}. \qquad\qquad (3.1.6)$$

Inserting Eq. (3.1.6) with its essential derivatives and Eq. (2.2.6) into Eq. (3.1.5) makes a polynomial in $(D_\xi^\alpha G(\xi)/G^2(\xi))$. Collect each coefficient of this polynomial and set to zero, we get a system of algebraic equations for $\iota_0, \iota_1, \tau_0, \tau_1, r$ and k whose values are obtained by solving these equations by computational software Maple as follows:

Set 1: $\iota_0 := \pm i\mu\tau_1; \, \iota_1 := 0; \, \tau_0 := 0; \, k := -\frac{r^2}{4} + 2\mu\lambda;$ $\qquad\qquad (3.1.7)$

Set 2: $\iota_0 := \mp i\mu\tau_1; \, \iota_1 := \pm i\tau_0\lambda; \, k := -\frac{r^2}{4} + 2\mu\lambda;$ $\qquad\qquad (3.1.8)$

Inserting the values from Eq. (3.1.7) and Eq. (3.1.8) into Eq. (3.1.6) yields

$$u_1(\xi) = \frac{\pm i\mu e^{i\varphi}}{(D_\xi^\alpha G/G^2)}, \qquad\qquad (3.1.9)$$

$$u_2(\xi) = e^{i\varphi} \times \frac{\mp i\mu\tau_1 \pm it\tau_0\lambda\left(D_\xi^\alpha G/G^2\right)}{\tau_0 + \tau_1\left(D_\xi^\alpha G/G^2\right)}, \tag{3.1.10}$$

Eqs. (3.1.9), (3.1.10) together with the Eqs. (2.2.8)-(2.2.10) make available the following outcomes:

$$u_1^{1,2}(\xi) = \pm e^{i\varphi}\sqrt{\frac{\lambda}{\mu}} \times \frac{i\mu \times (Bcos\sqrt{\lambda\mu}\xi^\alpha/\alpha - Asin\sqrt{\lambda\mu}\xi^\alpha/\alpha)}{Acos\sqrt{\lambda\mu}\xi^\alpha/\alpha + Bsin\sqrt{\lambda\mu}\xi^\alpha/\alpha}, \lambda\mu > 0, \tag{3.1.11}$$

where $\varphi = (-r/2)^{1/\alpha}x + \left((8\lambda\mu - r^2)/4\right)^{1/\alpha}t, \xi = x + r^{1/\alpha}t$.

$$u_1^{3,4}(\xi) = \mp e^{i\varphi}\sqrt{\frac{\lambda}{|\lambda\mu|}} \times \frac{i\mu \times \{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) - B\}}{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + B}, \lambda\mu < 0, \tag{3.1.12}$$

where $\varphi = (-r/2)^{1/\alpha}x + \left((8\lambda\mu - r^2)/4\right)^{1/\alpha}t, \xi = x + r^{1/\alpha}t$.

$$u_2^{5,6}(\xi) = e^{i\varphi} \times \frac{\mp i\mu\tau_1 \pm it\tau_0\sqrt{\lambda\mu} \times \frac{Acos\sqrt{\lambda\mu}\xi^\alpha/\alpha + Bsin\sqrt{\lambda\mu}\xi^\alpha/\alpha}{Bcos\sqrt{\lambda\mu}\xi^\alpha/\alpha - Asin\sqrt{\lambda\mu}\xi^\alpha/\alpha}}{\tau_0 + \tau_1\sqrt{\mu/\lambda} \times \frac{Acos\sqrt{\lambda\mu}\xi^\alpha/\alpha + Bsin\sqrt{\lambda\mu}\xi^\alpha/\alpha}{Bcos\sqrt{\lambda\mu}\xi^\alpha/\alpha - Asin\sqrt{\lambda\mu}\xi^\alpha/\alpha}}, \lambda\mu > 0, \tag{3.1.13}$$

where $\varphi = (-r/2)^{1/\alpha}x + \left((8\lambda\mu - r^2)/4\right)^{1/\alpha}t, \xi = x + r^{1/\alpha}t$.

$$u_2^{7,8}(\xi) = e^{i\varphi} \times \frac{\mp i\mu\tau_1 \mp it\tau_0\sqrt{\lambda|\lambda\mu|} \times \frac{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + B}{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) - B}}{\tau_0 - \tau_1\sqrt{|\lambda\mu|/\lambda} \times \frac{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + B}{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) - B}}, \lambda\mu < 0, \tag{3.1.14}$$

where $\varphi = (-r/2)^{1/\alpha}x + \left((8\lambda\mu - r^2)/4\right)^{1/\alpha}t, \xi = x + r^{1/\alpha}t$.

$$u_2^{9,10}(\xi) = e^{i\varphi} \times \frac{\mp it\tau_0 \times \frac{A\alpha}{(A\xi^\alpha + B\alpha)}}{\tau_0 - \tau_1 \times \frac{A\alpha}{\lambda(A\xi^\alpha + B\alpha)}}, \mu = 0, \lambda \neq 0, \tag{3.1.15}$$

where $\varphi = (-r/2)^{1/\alpha}x + \left((8\lambda\mu - r^2)/4\right)^{1/\alpha}t, \xi = x + r^{1/\alpha}t$.

The above obtained solutions are compared with those of (Kaplan et al., 2016) and seemed to be different which first time recorded in the literature.

3.2. The (2+1)-dimensional time-fractional Schrodinger equation

The (2+1)-dimensional time-fractional Schrodinger equation is

$$iD_t^\alpha u + \rho u_{xx} - \eta u_{yy} + \tau u|u|^2 = 0, \tag{3.2.1}$$

where ρ, η and τ are unknown parameters.

The consideration

$$u(x, y, t) = u(\xi), \xi = x + y + r^{1/\alpha}t, \tag{3.2.2}$$

forces Eq. (3.2.1) to become

$$irD_\xi^\alpha u + \rho D_\xi^{2\alpha}u - \eta D_\xi^{2\alpha}u + \tau u|u|^2 = 0. \tag{3.2.3}$$

with

$$u(\xi) = e^{ik\xi}v(\xi), \tag{3.2.4}$$

where $v(\xi)$ is a real-valued function and k is unknown parameter. Equating real and imaginary parts from Eq. (3.2.3) yields

$$rD_\xi^\alpha v + 2k(\rho - \eta)D_\xi^\alpha v = 0, \tag{3.2.5}$$

$$(\rho - \eta)D_\xi^{2\alpha} v + \tau v^3 - \left(kr + k^2(\rho - \eta)\right)v = 0. \tag{3.2.6}$$

From Eq. (3.2.5),

$$k = \frac{r}{2(\eta - \rho)}. \tag{3.2.7}$$

Eq. (3.2.6) reduces to

$$4(\rho - \eta)^2 D_\xi^{2\alpha} v + 4\tau(\rho - \eta)v^3 + r^2 v = 0. \tag{3.2.8}$$

Due to the homogenous balance principle, we attain $n = 1$ and solution (2.2.4) turns into

$$v(\xi) = \frac{\iota_0 + \iota_1(D_\xi^\alpha G(\xi)/G^2(\xi))}{\tau_0 + \tau_1(D_\xi^\alpha G(\xi)/G^2(\xi))}. \tag{3.2.9}$$

Eq. (3.2.8) with solution (3.2.9) and Eq. (2.2.5) becomes a polynomial in $(D_\xi^\alpha G(\xi)/G^2(\xi))$. Set each coefficient to zero and solve by computational software Maple for the following outcomes:

Set 1: $\iota_0 := 0; \iota_1 := \pm\sqrt{\frac{2(\eta - \rho)}{\sigma}}\tau_0\lambda; \tau_1 = 0; r = \pm 2\sqrt{-2\lambda\mu}(\eta - \rho);$ \hfill (3.2.10)

Set 2: $\iota_0 := \pm\sqrt{\frac{2(\eta - \rho)}{\sigma}}\tau_1\mu; \iota_1 := \mp\sqrt{\frac{2(\eta - \rho)}{\sigma}}\tau_0\lambda; r = \pm 2\sqrt{-2\lambda\mu}(\eta - \rho);$ \hfill (3.2.11)

Operating the values available in Eqs. (3.2.10)-(3.2.11) into Eq. (3.2.9) yields the following expressions for chosen solutions:

$$u_1(\xi) = \pm e^{ik\xi} \times \sqrt{\frac{2(\eta - \rho)}{\sigma}}\lambda\left(D_\xi^\alpha G/G^2\right), \tag{3.2.13}$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

$$u_2(\xi) = e^{ik\xi} \times \frac{\sqrt{\frac{2(\eta - \rho)}{\sigma}}\left\{\pm\tau_1\mu\mp\tau_0\lambda\left(D_\xi^\alpha G/G^2\right)\right\}}{\tau_0 + \tau_1\left(D_\xi^\alpha G/G^2\right)}, \tag{3.2.14}$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

Eq. (3.2.13) and Eq. (3.2.14) along with Eqs. (2.2.8)-(2.2.10) provide the following exact solutions:

$$u_1^{1,2}(\xi) = \pm e^{ik\xi}\sqrt{\frac{2\mu\lambda(\eta - \rho)}{\sigma}} \times \frac{A\cos\sqrt{\lambda\mu}\xi^\alpha/\alpha + B\sin\sqrt{\lambda\mu}\xi^\alpha/\alpha}{B\cos\sqrt{\lambda\mu}\xi^\alpha/\alpha - A\sin\sqrt{\lambda\mu}\xi^\alpha/\alpha}, \lambda\mu > 0, \tag{3.2.15}$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

$$u_1^{3,4}(\xi) = \mp e^{ik\xi}\sqrt{\frac{2\lambda|\lambda\mu|(\eta - \rho)}{\sigma}} \times \frac{A\sinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + A\cosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + B}{A\sinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) + A\cosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha) - B}, \lambda\mu < 0, \tag{3.2.16}$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

$$u_1^{5,6}(\xi) = \mp e^{ik\xi}\sqrt{\frac{2(\eta - \rho)}{\sigma}} \times \frac{A\alpha}{A\xi^\alpha + B\alpha}, \mu = 0, \lambda \neq 0, \tag{3.2.17}$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

$$u_2^{7,8}(\xi) = e^{ik\xi} \times \frac{\sqrt{\frac{2(\eta-\rho)}{\sigma}}\left\{\pm\tau_1\mu\mp\tau_0\sqrt{\lambda\mu}\times\frac{Acos\sqrt{\lambda\mu}\xi^\alpha/\alpha+Bsin\sqrt{\lambda\mu}\xi^\alpha/\alpha}{Bcos\sqrt{\lambda\mu}\xi^\alpha/\alpha-Asin\sqrt{\lambda\mu}\xi^\alpha/\alpha}\right\}}{\tau_0+\tau_1\sqrt{\mu/\lambda}\times\frac{Acos\sqrt{\lambda\mu}\xi^\alpha/\alpha+Bsin\sqrt{\lambda\mu}\xi^\alpha/\alpha}{Bcos\sqrt{\lambda\mu}\xi^\alpha/\alpha-Asin\sqrt{\lambda\mu}\xi^\alpha/\alpha}}, \lambda\mu > 0, \qquad (3.2.18)$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

$$u_2^2(\xi) = e^{ik\xi} \times \frac{\sqrt{\frac{2(\eta-\rho)}{\sigma}}\left\{\pm\tau_1\mu\pm\tau_0\sqrt{\lambda|\lambda\mu|}\times\frac{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)+Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)+B}{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)+Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)-B}\right\}}{\tau_0-\tau_1\sqrt{|\lambda\mu|/\lambda}\times\frac{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)+Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)+B}{Asinh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)+Acosh(2\sqrt{\lambda\mu}\xi^\alpha/\alpha)-B}}, \lambda\mu < 0, \qquad (3.2.19)$$

where $= \pm\sqrt{-2\lambda\mu}$, $\xi = x + y + \left(\pm 2\sqrt{-2\lambda\mu}(\eta - \rho)\right)^{1/\alpha} t.$

The above achieved solutions are compared with the existing results in the literature which shown the novelty of our results (Rizvi et al., 2017).

GRAPHICAL REPRESENTATIONS OF THE OBTAINED SOLUTIONS

The solutions furnished above to the suggested equations are brought out graphically for their physical appearances. The outlined solitons take different shape such as kink, bell, cuspon, peakon, periodic etc. For simplicity, we display few graphs as follows:
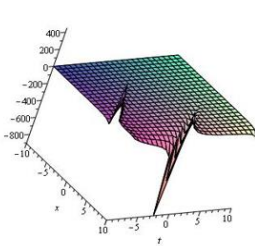

Figure 1: The plot of solution (3.1.11) for $\alpha = 1, \beta = 1, r = 1, \lambda = 3, \mu = .02, m = .5, A = 3, B = 1$ within $-10 \leq x, t \leq 10.$
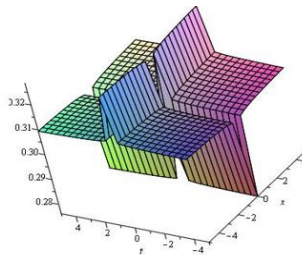
Figure 2: The outline of solution (3.1.12) for $\alpha = .001, \mu = -.02$, $\beta = .001, r = .002, \lambda = .04,$　m = .003, A = 3 , B = 1 within $-5 \leq x, t \leq 5.$
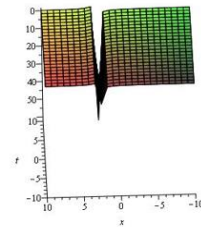
Figure 3: The shape of solution (3.1.13) for $\alpha = \beta = B = 1, \lambda = 2, r = 0.003, \tau_1 = 0.01,$ $m = \mu = 0.005, \tau_0 = 0.05, A = 4$ in $-10 \leq x, t \leq 10.$
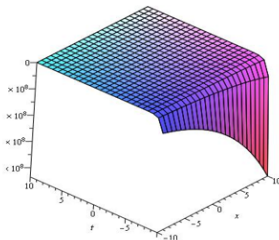

Figure 4: The shape of solution (3.1.14) for $\alpha = 1, \beta = 1, r = 3, \lambda = 0.2, \mu = -.05, \tau_0 = 0.2, \tau_1 = 0.1, m = 5, A = 4, B = 1$ within $-10 \leq x, t \leq 10.$
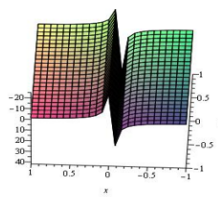
Figure 5: The singular king shape soliton of solution (3.1.15) for $\alpha = 1, \beta = 1, r = .01, \lambda = 0.02, \tau_0 = 0.05, \tau_1 = 0.01, m = 0.3, A = 4, B = 1$ within $-1 \leq x, t \leq 1.$
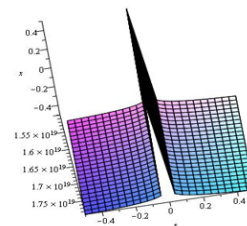
Figure 6: The anti-bell shape soliton of solution (3.2.15) for $\alpha = 0.0001, \lambda = 2, \mu = 0.01, \eta = 0.04, \rho = 0.1, \sigma = 2, y = 0, A = 3, B = 1$ within $-0.5 \leq x, t \leq 0.5.$
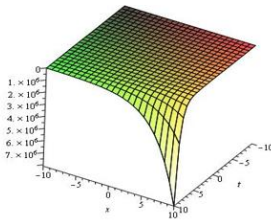
Figure 7: The shape of solution (3.2.17) for $\alpha = 1, \lambda = 0.2, \mu = -0.3, \eta = 5, \sigma = .002, \rho = .0001, A = 3, B = 2, y = 0$ within $-10 \leq x, t \leq 10$.
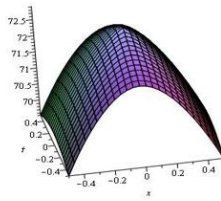
Figure 8: The plot of solution (3.2.18) for $\alpha = 1, \lambda = 0.2, \mu = 0.3, \eta = -0.4, \sigma = 0.04, \rho = 0.2, A = 3, B = 2, y = 0, \tau_0 = 0.02, \tau_1 = 0.03$ within $-0.5 \leq x, t \leq 0.5$.
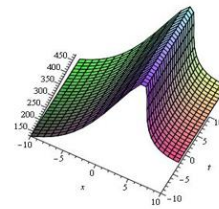
Figure 9: Shape of (3.2.19) for $\alpha = 1, \lambda = 0.02,$ $A = 3,$ $\mu = -0.03,$

$B = 2, \eta = 0.4, \sigma = 5, \rho = 0.2,$

$\tau_1 = 0.003,$ $y = 0, \tau_0 = 0.002,$ $-10 \leq x, t \leq 10$.

CONCLUSIONS

We aim to construct exact analytic solutions to the space-time fractional nonlinear Schrodinger equation and the (2+1)-dimensional time-fractional nonlinear Schrodinger equation by making use of the recently established rational $(D_\xi^\alpha G/G^2)$-expansion method. Consequently, three type's soliton solutions such as hyperbolic function, trigonometric function and rational function have been gained which are different from those of literature. The obtained solutions are outlined graphically to depict nonlinear complex physical phenomena arise in applied mathematics, mathematical physics and engineering. The performance of the employed method is claimed to be used for further research relating to the nonlinear models.

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