A Green Integrated Inventory Model for a Three-Tier Supply Chain of an Agricultural Product

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ABSTRACT

Green supply chain management coordinates environment issues into the supply chain business. It has been popular to both academicians and practitioners. Smooth supply of processed agricultural products is essential for human beings and pets. In some models, excess raw materials, byproducts and defected products are kept neglected in producing and marketing finished products. Here, we have presented a three-tier green supply chain model for an agricultural product where byproducts are used for some purposes. Solution procedure of the model is derived. We have demonstrated the model using two numerical example problems.

Key words:
Agricultural product, green supply chain, integrated inventory, lot size, supply chain, byproduct

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INTRODUCTION

Controlling of environmental pollution is a burning issue in the universe. Usually, a green supply chain (SC) uses the mechanism to reduce environmental pollution during its functioning. So, the topic has got the attention of researchers worldwide. We are also interested of modeling a green supply chain problem with coordinated inventory policy. Current competitive market imposes companies to integrate with their upstream and downstream players for establishing an improved SC to minimize the management cost.

They all supply finished products to customers with reasonable prices (e.g., Gunasekaran et al. 2008; Ben-Daya and Al-Nassar 2008; Moncayo-Martı ́nez and Zhang 2013; Islam and Hoque 2014a; Hellion et al. 2015; Islam and Hoque 2018). Modeling on integrated cost minimization using joint economic lot sizing (JELS) policy has received the devotion of the researchers and also the practitioners in SC business (Banerjee 1986; Baboli et al. 2011; Glock 2012; Wang et al. 2015; Sarmah et al. 2006). Most of the studies related to JELS approach are conducted within the context of a two-tier SC consisting of a vendor and a buyer (Kim et al. 2014; Giri and Bardhan 2015; Hariga et al. 2016). In a two stage SC, the manufacturer on receiving an order produces plenty in one setup, and delivers them to the buyers with a number of shipments to minimize the chain wide cost (Lee 2005; Hoque 2011; Sari et al. 2012).

At the initial stage of JELS study, Goyal (1977) proposed a lot sizing policy for two stage production. Then, considering numerous techniques of supply chain synchronization like equal-sized shipments, unequal sized shipments, combination of equal and unequal sized shipments, etc., researchers established JELS policy as a beneficial tool in SC management (Hill 1999; Goyal and Nebebe 2000; Ben-Daya and Hariga 2004; Ben-Daya et al. 2008). Khouja (2003) extended the JELS policy to a three-tier SC for supplying products from a vendor to several customers. Here, the cycle time at each stage was the integer multiple of the cycle time of the adjacent downstream player. Ben-Daya et al. (2013), and Abdelsalam and Elassal (2014) improved that model by considering common cycle time.

However, the setting to process of agricultural products is somewhat different. In case of processing an agricultural product, essential supplying period for the supplier cannot be an integer multiple of the cycle time of the manufacturer or of the retailer(s). Rather it is a fraction of the manufacturer's cycle time because the period of harvesting an agricultural product is shorter than the retailing period of a finished product (Islam 2014; Islam and Hoque 2014b; Gal et al. 2008; Ca´rdenas-Barro´n 2012; Islam et al. 2017; Ca´rdenas-Barro´n et al. 2012). There is a scarcity of a three-tier model involving agricultural products within the existing literature. Islam and Hoque (2017) developed a three-tier model of processing an agricultural product, considering raw material supplying period for the supplier as a fraction of the manufacturer's cycle time (retailing period). Here, JELS policy is utilized to minimize the integrated cost of a cycle period for the proposed supply chain.

In integrated production and inventory decision models, players should realize the chain objectives with the coordinated decision process. Coordinated multi-level productions and inventories are addressed well in the literatures of Ben-Daya and Al-Nassar 2008; Chen and Chen 2005; Chung et al. 2008; Jaber and Goyal 2008; Khouja 2003; Munson and Rosenblatt 2001; Sarmah et al. 2006; Islam and Hoque 2019, Islam et al. 2020. Based on a literature review which has been carried out, a few researches are concerned with the whole manufacturing system, that is, they overlook the defected products of the system. We have proposed here a green supply chain model that considers also the byproducts of the system.

To make the model reader friendly, we have organized the paper as follows. Section 2 defines the problem and describes assumptions and notation. Derivation of the mathematical model is given in Section 3, while the solution technique and algorithm are provided in Section 4. The model is analyzed by numerical example problems in Section. 5. Finally, Section 6 concludes by highlighting the study findings, limitations and future research scopes.

Problem Definition, Assumptions and Notation

Figure 1 provides a flowchart of the raw materials, finished products, byproducts of our three-tier whole green manufacturing supply chain.

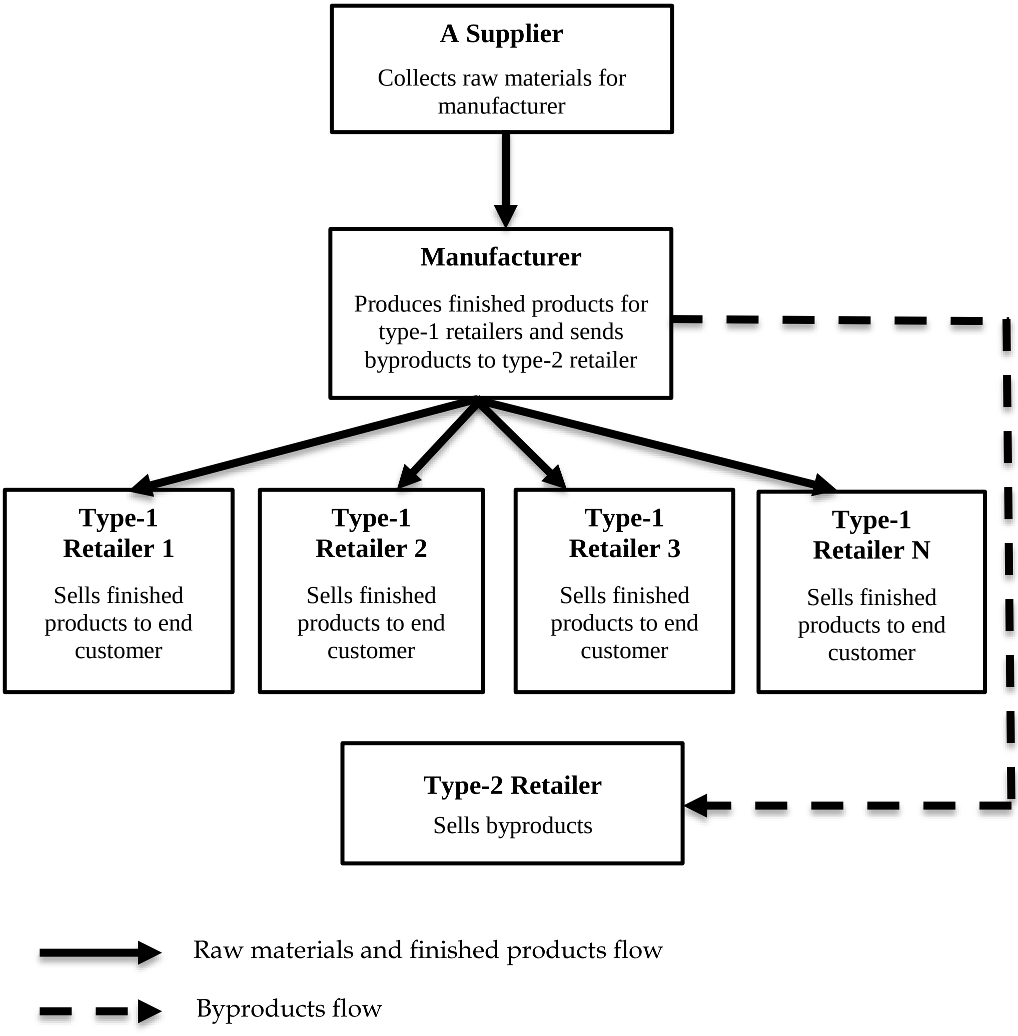

Figure 1: Scenarios of this three-tier supply chain problem

After getting a supply order, a supplier, within a short harvesting period, supplies raw materials to Manufacturer. Manufacturer produces finished products and delivers them to type 1 retailers to fulfil their orders. The manufacturer also sends byproducts to Type 2 retailer. The model uses targeted products and byproducts and hence, the environment keeps green. To avoid shortage at any stage, we assume that demand rate of the downstream players is less than or equal to production rate of the upstream players. The following notation are used in developing the model.

Notation

P_s collection rate of the supplier.

P_m production rate of the Manufacturer.

C_m conversion rate of raw materials to finished product.

N number of type 1 retailers.

D_r demand rate incurred by type 1 retailer r ($D = \sum_{r=1}^{N} D_r$).

D_{r_2} demand rate incurred by type 2 retailer.

D_m demand rate of the Manufacturer, $D_m = \frac{D}{C_m}$.

T cycle time of the retailers, the manufacturer and the supplier.

M_1 number of shipments in a cycle received by the Manufacturer.

M_2 number of shipments in a cycle received by a type 1 retailer.

M_3 number of shipments in a cycle received by a type 2 retailer.

A_m Manufacturer's production setup cost.

O_m Manufacturer's raw item ordering cost.

O_r type 1 retailer's ordering cost.

O_{r_2} type 2 retailer's ordering cost.

O_s supplier's raw material order cost.

S_m cost per shipment from the supplier to the Manufacturer.

S_r cost per shipment from Manufacturer to the type 1 retailer r.

S_{r_2} cost per shipment from Manufacturer to the type 2 retailer.

H_s holding cost per unit time for the supplier.

H_m per unit raw material holding cost for the Manufacturer per unit time.

H_f per unit finished product holding cost for the Manufacturer per unit time.

H_b per unit byproduct holding cost for the Manufacturer per unit time.

H_r per unit holding cost for type 1 retailers per unit time.

H_{r_2} per unit holding cost for the type 2 retailers per unit time.

TC entire supply chain cost per unit time.

MATHEMATICAL MODEL DEVELOPMENT

Based on the situation described above, and with the underpinning, assumptions and notation, we formulate the cost components of all players involved in the supply chain. The integrated model of the described problem is presented below.

Type-1 retailers experience only the cost of order (O_r/T), transportation (M_2S_r/T) and holding $(H_rTD_r/2M_2)$. Thus, the total cost per unit time for Type-1 retailers is

$$TC_1 = \sum_{r=1}^{N}\left(\frac{O_r}{T} + \frac{M_2S_r}{T} + H_r\frac{TD_r}{2M_2}\right). \quad (1)$$

As Type-1 retailers, Type-2 retailer incurs the cost of order (O_{r_2}/T), transportation $(M_3S_{r_2}/T)$ and byproduct holding $(H_{r_2}TD_{r_2}/2M_3)$. Therefore, the total cost per unit time for Type-2 retailers is

$$TC_2 = \frac{O_{r_2}}{T} + \frac{M_3S_{r_2}}{T} + H_{r_2}\frac{TD_{r_2}}{2M_3}. \quad (2)$$

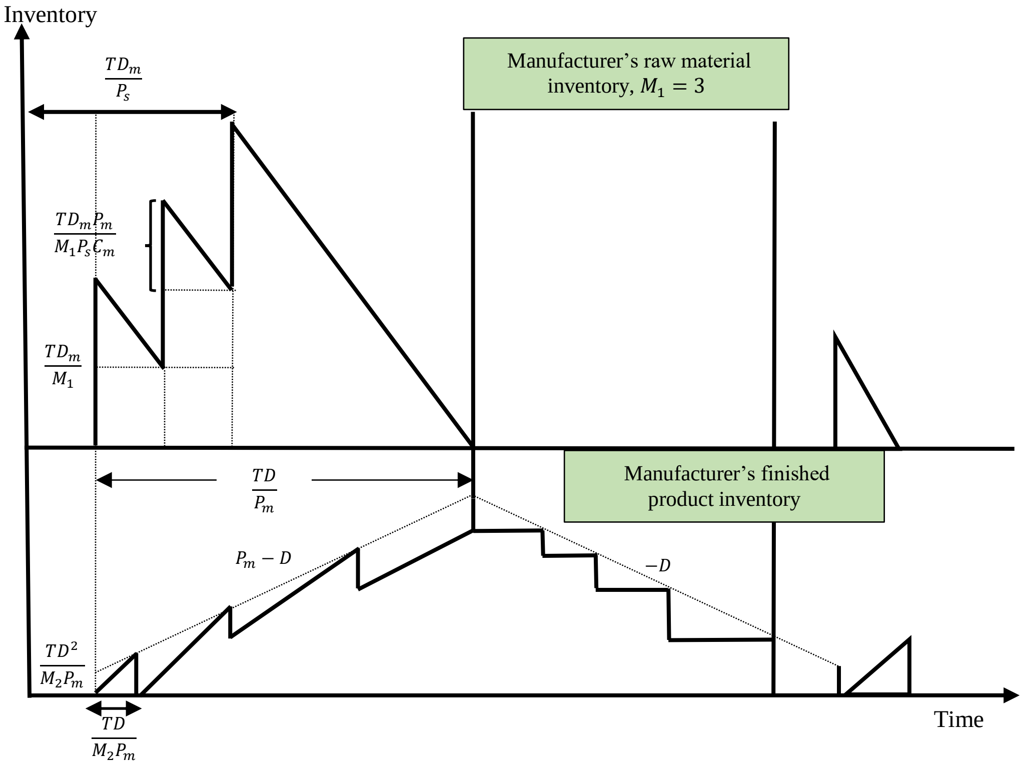

Figure 2: On-hand inventories of the manufacturer

Manufacturer bears the cost of raw material, order (O_m/T), raw material shipment (M_1S_m/T), raw material holding, setup (A_m/T), finished product and byproduct holding. $(\frac{TD_m}{M_1} - \frac{TD_mP_m}{M_1P_sC_m})$ units of raw material are accumulated to the manufacturer from each shipment, which is depicted in the diagram of 'Manufacturer's raw material inventory' in Figure 2. After M_1-th shipment, raw material inventory gradually decreases to zero in producing finished products. Thus, the raw material inventory (RI_m) per unit time is as given below.

$$RI_m = \frac{1}{T}\Big[\frac{1}{2}\frac{TD_m}{M_1P_s}.\frac{TD_mP_m}{M_1P_sC_m} \times (M_1 - 1) + \frac{1}{2}\Big\{(M_1 - 1)\left(\frac{TD_m}{M_1} - \frac{TD_mP_m}{M_1P_sC_m}\right) + \frac{TD_m}{M_1}\Big\}.\Big\{\frac{TD_m}{P_m} - (M_1 - 1)\frac{TD_m}{M_1P_s}\Big\} + \frac{TD_m}{M_1P_s}\left(\frac{TD_m}{M_1} - \frac{TD_mP_m}{M_1P_sC_m}\right)\{1 + 2 + 3 + \cdots + (M_1 - 1)\}\Big],$$

$$RI_m = \frac{TD_m^2}{2}\left(\frac{1}{P_m} + \frac{1}{M_1P_s} - \frac{1}{C_mP_s}\right).$$

Therefore, the raw material holding cost (HC_m) per unit time for the manufacturer is

$$HC_m = H_m \frac{TD_m^2}{2}\left(\frac{1}{P_m} + \frac{1}{M_1 P_s} - \frac{1}{C_m P_s}\right) \quad (3)$$

The system inventory (inventories of manufacturer and retailers) of finished product is depicted by the dashed line in 'Manufacturer's finished product inventory' in Figure 2. This inventory increases from TD^2/M_2P_m (kept by Type-1 retailers), by the rate of $P_m - D$ during the production period TD/P_m. Then it decreases at a rate of D in meeting demand of the retailers up to the end of the cycle. Thus, the average system inventory is

$$(P_m - D)\frac{TD}{2P_m} + \frac{TD^2}{M_2 P_m}.$$

Retailers' average inventory $TD/2M_2$ is also included in the average system inventory. Thus, the manufacturer's finished product inventory holding cost (HC_f) per unit time is

$$HC_f = H_f\left[(P_m - D)\frac{TD}{2P_m} + \frac{TD^2}{M_2 P_m} - \frac{TD}{2M_2}\right]. \quad (4)$$

As above, the system inventory of byproduct increases from $\frac{TD_{r_2}^2}{M_3(D_m - D)}$ (kept by Type-2 retailer), by the rate of $(D_m - D) - D_{r_2}$ during the production period $\frac{TD_{r_2}}{(D_m - D)}$ or TD/P_m. Then it decreases at a rate of D_{r_2} in meeting demand of Type-2 retailer up to the end of the cycle. Hence, the average system inventory of byproduct is

$$\left((D_m - D) - D_{r_2}\right)\frac{TD_{r_2}}{2(D_m - D)} + \frac{TD_{r_2}^2}{M_3(D_m - D)}.$$

Average inventory $(\frac{TD_{r_2}}{2M_3})$ of Type 2 retailer is also included in the average system inventory above. Thus, the manufacturer's byproduct inventory holding cost (HC_d) per unit time is

$$HC_d = H_b\left[\left((D_m - D) - D_{r_2}\right)\frac{TD_{r_2}}{2(D_m - D)} + \frac{TD_{r_2}^2}{M_3(D_m - D)} - \frac{TD_{r_2}}{2M_3}\right].$$

Hence, the manufacturer's total cost (TC_3) per unit time is given by

$$TC_3 = \frac{O_m}{T} + \frac{M_1 S_m}{T} + \frac{A_m}{T} + H_m \frac{TD_m^2}{2}\left(\frac{1}{P_m} + \frac{1}{M_1 P_s} - \frac{1}{C_m P_s}\right) + H_f\left[(P_m - D)\frac{TD}{2P_m} + \frac{TD^2}{M_2 P_m} - \frac{TD}{2M_2}\right] + H_b\left[\left((D_m - D) - D_{r_2}\right)\frac{TD_{r_2}}{2(D_m - D)} + \frac{TD_{r_2}^2}{M_3(D_m - D)} - \frac{TD_{r_2}}{2M_3}\right] \quad (5)$$

Supplier incurs only the cost of ordering, O_s/T, and holding, $H_sT{D_m}^2/2M_1P_s$, because the manufacturer takes TD_m/M_1 units of raw material from the supplier in each shipment. So, the total cost incurred by the supplier is

$$TC_4 = \frac{O_s}{T} + H_s \frac{TD_m^2}{2M_1 P_s}. \quad (6)$$

Thus, the total supply chain cost is simply the sum of the costs experienced by Type-1 retailers, Type-2 retailer, manufacturer and supplier. Hence, it is found by adding Equations (1), (2), (5) and (6) as follows.

$$TC(M_1, M_2, M_3) = \frac{1}{T}\sum_{r=1}^{N} O_r + \frac{M_2}{T}\sum_{r=1}^{N} S_r + \frac{T}{2M_2}\sum_{r=1}^{N} H_r D_r + \frac{O_{r_2}}{T} + \frac{M_3 S_{r_2}}{T} + \frac{H_{r_2} T D_{r_2}}{2M_3} + \frac{O_m}{T} + \frac{M_1 S_m}{T} + \frac{A_m}{T} + \frac{H_m T D_m^2}{2P_m} + \frac{H_m T D_m^2}{2M_1 P_s} - \frac{H_m T D_m^2}{2C_m P_s} + \frac{H_f T D}{2} - \frac{H_f T D^2}{2P_m} + \frac{H_f T D^2}{M_2 P_m} - \frac{H_f T D}{2M_2} + \frac{H_b T D_{r_2}}{2} - \frac{H_b T D_{r_2}^2}{2(D_m - D)} + \frac{H_b T D_{r_2}^2}{M_3(D_m - D)} - \frac{H_b T D_{r_2}}{2M_3} + \frac{O_s}{T} + \frac{H_s T D_m^2}{2M_1 P_s} \quad (7)$$

It is supposed to minimize the total supply chain cost in (7) based on the values of M_1, M_2, M_3.

DEVELOPMENT OF SOLUTION TECHNIQUE

We have to minimize the integrated cost function (7) with respect to the integer variables M_1 , M_2 *and* M_3. The model is solved using the calculus method of optimization. To obtain integer solutions to the variables M_1 , M_2 *and* M_3 for integrated minimal total cost, we use parallel multiple jumps technique as used by (e.g., Abdelsalam and Elassal 2014; Ca´rdenas-Barro´n et al. 2012; Islam and Hoque 2017). We reorganize the total cost function (7) as follows,

$$TC(M_1, M_2, M_3) = \frac{1}{M_1}\left[M_1{}^2\frac{S_m}{T} + M_1\left\{\frac{1}{M_2}\left(M_2{}^2\frac{1}{T}\sum_{r=1}^{N} S_r + M_2\left(\frac{1}{T}\sum_{r=1}^{N} O_r + \frac{O_m}{T} + \frac{A_m}{T} + \frac{O_s}{T} + \frac{H_mTD_m^2}{2P_m} + \frac{H_fTD}{2} - \frac{H_mTD_m^2}{2C_mP_s} - \frac{H_fTD^2}{2P_m}\right) + \left(\frac{T}{2}\sum_{r=1}^{N} H_r D_r + \frac{H_fTD^2}{P_m} - \frac{H_fTD}{2}\right)\right)\right\} + \left(\frac{H_mTD_m^2}{2P_s} + \frac{H_sTD_m^2}{2P_s}\right)\right] + \frac{1}{M_3}\left[\ M_3{}^2\ \frac{S_{r_2}}{T} + M_3\left\{\frac{O_{r_2}}{T} + \frac{H_bTD_{r_2}}{2} - \frac{H_bTD_{r_2}^2}{2(D_m-D)}\right\} + \left\{\frac{H_{r_2}TD_{r_2}}{2} + \frac{H_bTD_{r_2}^2}{(D_m-D)} - \frac{H_bTD_{r_2}}{2}\right\}\right]. \quad (8)$$

Denoting, $\gamma = \frac{S_m}{T}$, $\beta = \frac{H_mTD_m^2}{2P_s} + \frac{H_sTD_m^2}{2P_s}$, $\theta = \frac{1}{T}\sum_{r=1}^{N} O_r + \frac{O_m}{T} + \frac{A_m}{T} + \frac{O_s}{T} + \frac{H_mTD_m^2}{2P_m} + \frac{H_fTD}{2} - \frac{H_mTD_m^2}{2C_mP_s} - \frac{H_fTD^2}{2P_m}$, $\varphi = \frac{T}{2}\sum_{r=1}^{N} H_r D_r + \frac{H_fTD^2}{P_m} - \frac{H_fTD}{2}$, $\varepsilon = \frac{1}{T}\sum_{r=1}^{N} S_r$, $\sigma = \frac{O_{r_2}}{T} + \frac{H_bTD_{r_2}}{2} - \frac{H_bTD_{r_2}^2}{2(D_m-D)}$, $\mu = \frac{H_{r_2}TD_{r_2}}{2} + \frac{H_bTD_{r_2}^2}{(D_m-D)} - \frac{H_bTD_{r_2}}{2}$, $\delta = \frac{S_{r_2}}{T}$, we have,

$$TC(M_1, M_2, M_3) = \frac{1}{M_1}\left[M_1{}^2\gamma + M_1\left\{\frac{1}{M_2}(M_2{}^2\varepsilon + M_2\theta + \varphi)\right\} + \beta\right] + \frac{1}{M_3}\left[(M_3{}^2\delta + M_3\sigma) + \mu\right]. \quad (9)$$

Considering the necessary condition, $\frac{\partial}{\partial M_1}(TC) = 0$, $\frac{\partial}{\partial M_2}(TC) = 0$ and $\frac{\partial}{\partial M_3}(TC) = 0$ for minimization of $TC(M_1, M_2, M_3)$ in (9), and hence we have

$$M_1 = \sqrt{\frac{D_m^2T^2(H_m+H_s)}{2P_sS_m}} = \sqrt{\frac{\beta}{\gamma}}, \quad (10)$$

$$M_2 = \sqrt{\frac{T}{\sum_{r=1}^{N} S_r}\left(\frac{T\sum_{r=1}^{N} H_rD_r}{2} + \frac{H_fTD^2}{P_m} - \frac{H_fTD}{2}\right)} = \sqrt{\frac{\varphi}{\varepsilon}}, \quad (11)$$

$$M_3 = \sqrt{\frac{T}{S_{r_2}}\left(\frac{H_{r_2}TD_{r_2}}{2} - \frac{H_bTD_{r_2}}{2} + \frac{H_bTD_{r_2}^2}{D_m-D}\right)} = \sqrt{\frac{\mu}{\delta}}. \quad (12)$$

The critical values in Equations (10), (11) and (12) are the optimal values of M_1 , M_2 *and* M_3 respectively because

$$\frac{\partial^2}{\partial M_1{}^2}(TC) = \frac{H_mTD_m^2}{M_1^3P_s} + \frac{H_sTD_m^2}{M_1^3P_s} > 0,$$

$$\frac{\partial^2}{\partial M_1{}^2}(TC)\frac{\partial^2}{\partial M_2{}^2}(TC) - \frac{\partial^2}{\partial M_1\partial M_2}(TC)\frac{\partial^2}{\partial M_2\partial M_1}(TC) = \left(\frac{H_mTD_m^2}{M_1^3P_s} + \frac{H_sTD_m^2}{M_1^3P_s}\right)\left(\frac{T}{M_2^3}\sum_{r=1}^{N} H_rD_r + \frac{2H_fTD^2}{M_2^3P_m} - \frac{H_fTD}{M_2^3}\right) > 0,$$

and

$$\frac{\partial^2}{\partial M_1{}^2}(TC)\frac{\partial^2}{\partial M_2{}^2}(TC)\frac{\partial^2}{\partial M_3{}^2}(TC) - \frac{\partial^2}{\partial M_1{}^2}(TC)\frac{\partial^2}{\partial M_2\partial M_3}(TC)\frac{\partial^2}{\partial M_3\partial M_2}(TC) = \left(\frac{H_mTD_m^2}{M_1^3P_s} + \frac{H_sTD_m^2}{M_1^3P_s}\right)\left(\frac{T}{M_2^3}\sum_{r=1}^{N} H_rD_r + \frac{2H_fTD^2}{M_2^3P_m} - \frac{H_fTD}{M_2^3}\right)\left(\frac{2H_bTD_{r_2}^2}{M_3^3(D_m-D)} - \frac{H_bTD_{r_2}}{M_3^3} + \frac{H_{r_2}TD_{r_2}}{M_3^3}\right) > 0.$$

The values of M_1 , M_2 *and* M_3 are rounded up to obtain their corresponding integral values. Following algorithm is the summarization of the parallel multiple jumps and the calculus method.

Solution Algorithm

Step 1 Initialize all given parameters.

Step 2 Calculate values of M_1, M_2 *and* M_3 using Eqs. (10), (11) and (12) respectively. If any of M_1, M_2 *and* M_3 is not real, then this problem is inconsistent and GOTO Step 8; otherwise, select the smallest integer greater than or equal to the obtained values of M_i (i = 1, 2) as the integral M_i.

Step 3 Calculate the total cost, TC using Eq. (9).

Step 4 Let TC_{old} is the value of TC.

Step 5 Use parallel multiple jumps technique on (M_1, M_2, M_3) as described below to obtain the minimal M_1, M_2 and M_3.

a. If M_1, M_2 *and* M_3 are greater than or equal to 2, then calculate the total cost, TC by using Eq. (9) with the following 26 jumps on (M_1, M_2, M_3). These are $(M_1 - 1, M_2 - 1, M_3 - 1)$, $(M_1 - 1, M_2, M_3 - 1)$, $(M_1 - 1, M_2 + 1, M_3 - 1)$, $(M_1, M_2 - 1, M_3 - 1)$, $(M_1, M_2, M_3 - 1)$, $(M_1, M_2 + 1, M_3 - 1)$, $(M_1 + 1, M_2 - 1, M_3 - 1)$, $(M_1 + 1, M_2, M_3 - 1)$, $(M_1 + 1, M_2 + 1, M_3 - 1)$, $(M_1 - 1, M_2 - 1, M_3)$, $(M_1 - 1, M_2, M_3)$, $(M_1 - 1, M_2 + 1, M_3)$, $(M_1, M_2 - 1, M_3)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2 - 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1 - 1, M_2 - 1, M_3 + 1)$, $(M_1 - 1, M_2, M_3 + 1)$, $(M_1 - 1, M_2 + 1, M_3 + 1)$, $(M_1, M_2 - 1, M_3 + 1)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2 - 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

b. If M_3 *and* M_1 are greater than or equal to 2, then calculate total cost, TC by using Eq. (9) with the following 17 jumps of (M_1, M_2, M_3). These are $(M_1 - 1, M_2, M_3 - 1)$, $(M_1 - 1, M_2 + 1, M_3 - 1)$, $(M_1, M_2, M_3 - 1)$, $(M_1, M_2 + 1, M_3 - 1)$, $(M_1 + 1, M_2, M_3 - 1)$, $(M_1 + 1, M_2 + 1, M_3 - 1)$, $(M_1 - 1, M_2, M_3)$, $(M_1 - 1, M_2 + 1, M_3)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1 - 1, M_2, M_3 + 1)$, $(M_1 - 1, M_2 + 1, M_3 + 1)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

c. If M_3 *and* M_2 are greater than or equal to 2, then calculate total cost, TC by using Eq. (9) with the following 17 jumps of (M_1, M_2, M_3). These are $(M_1, M_2 - 1, M_3 - 1)$, $(M_1, M_2, M_3 - 1)$, $(M_1, M_2 + 1, M_3 - 1)$, $(M_1 + 1, M_2 - 1, M_3 - 1)$, $(M_1 + 1, M_2, M_3 - 1)$, $(M_1 + 1, M_2 + 1, M_3 - 1)$, $(M_1, M_2 - 1, M_3)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2 - 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1, M_2 - 1, M_3 + 1)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2 - 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

d. If only M_3 is greater than or equal to 2, then calculate total cost, TC by using Eq. (9) with the following 11 jumps of (M_1, M_2, M_3). These are $(M_1, M_2, M_3 - 1)$, $(M_1, M_2 + 1, M_3 - 1)$, $(M_1 + 1, M_2, M_3 - 1)$, $(M_1 + 1, M_2 + 1, M_3 - 1)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

e. If M_1 *and* M_2 are greater than or equal to 2, then calculate the total cost, TC by using Eq. (9) with the following 17 jumps on (M_1, M_2, M_3). These are $(M_1 - 1, M_2 - 1, M_3)$, $(M_1 - 1, M_2, M_3)$, $(M_1 - 1, M_2 + 1, M_3)$, $(M_1, M_2 - 1, M_3)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2 - 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1 - 1, M_2 - 1, M_3 + 1)$, $(M_1 - 1, M_2, M_3 + 1)$, $(M_1 - 1, M_2 + 1, M_3 + 1)$, $(M_1, M_2 - 1, M_3 + 1)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2 - 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

f. If only M_1 is greater than or equal to 2, then calculate total cost, TC by using Eq. (9) with the following 11 jumps of (M_1, M_2, M_3). These are $(M_1 - 1, M_2, M_3)$, $(M_1 - 1, M_2 + 1, M_3)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1 - 1, M_2, M_3 + 1)$, $(M_1 - 1, M_2 + 1, M_3 + 1)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

g. If *only* M_2 is greater than or equal to 2, then calculate the total cost, TC by using Eq. (9) with the following 11 jumps on (M_1, M_2, M_3). These are $(M_1, M_2 - 1, M_3)$, $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2 - 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1, M_2 - 1, M_3 + 1)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2 - 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

h. If none of M_1, M_2 *and* M_3 are greater than or equal to 2, then calculate total cost, TC by using Eq. (9) with the following 7 jumps of (M_1, M_2, M_3). These are $(M_1, M_2 + 1, M_3)$, $(M_1 + 1, M_2, M_3)$, $(M_1 + 1, M_2 + 1, M_3)$, $(M_1, M_2, M_3 + 1)$, $(M_1, M_2 + 1, M_3 + 1)$, $(M_1 + 1, M_2, M_3 + 1)$ and $(M_1 + 1, M_2 + 1, M_3 + 1)$.

i. Find the minimum of the minimal total costs calculated for each jump and denote the corresponding TC by TC_{new}, and the associated values of M_1, M_2 *and* M_3 respectively by M'_1, M'_2 and M'_3.

Step 6 If (TC_{old} - TC_{new}) > 0, then replace the values of TC_{old}, M_1, M_2 *and* M_3 by the values of TC_{new}, M'_1, M'_2 and M'_3 respectively, and GOTO Step 5.

Step 7 Print TC_{new}, M'_1, M'_2 and M'_3 as the output.

Step 8 STOP.

NUMERICAL ILLUSTRATION

We set two numerical example problems to illustrate the developed model. We consider seven Type-1 retailers and one Type-2 retailer. Data of these problems are given in Table 1 and Table 2. We consider cycle time, T = 0.5 year and the conversion rate of raw material to finished product, C_m= 0.7 for both examples.

For continuous cases, Example 1 provides numbers of shipments 47.759, 33.8908 and 31.8501 for Type-1 retailers, Type-2 retailer and manufacturer respectively. The corresponding chain wide cost per year is 54,693.4. However, Example 2 gives numbers of shipments 4.72378, 14.9265 and 5.15388 for Type-1 retailers, Type-2 retailer and manufacturer respectively; and corresponding total supply chain cost per year is 16,117.8.

Table 1: Provided data for Example 1

	Setup cost	Order cost	Holding Cost (finished)	Holding cost (raw)	Holding cost (byprod.)	Cost of Each shipment	Demand rate	Collection/ Production rate
Type 1 Retailer 1		45	7			12	10,000	
Type 1 Retailer 2		45	7			12	15,000	
Type 1 Retailer 3		45	7			12	20,000	
Type 1 Retailer 4		45	7			12	30,000	
Type 1 Retailer 5		45	7			12	35,000	
Type 1 Retailer 6		45	7			12	40,000	
Type 1 Retailer 7		45	7			12	5000	
Type 2 Retailer		30			5	8	18,000	
Manufacturer	150	170	4	0.5	2	25	221,429	180,000
Supplier		550		0.7			221,429	290,000

Table 2: Provided data for Example 2

	Setup cost	Order cost	Holding cost (finished)	Holding Cost (raw)	Holding Cost (byprod.)	Cost of Each shipment	Demand rate	Collection/ Production rate
Type 1 Retailer 1		105	4			20	1500	
Type 1 Retailer 2		110	4.5			22	1500	
Type 1 Retailer 3		115	5			23	1500	
Type 1 Retailer 4		120	5.5			25	1500	
Type 1 Retailer 5		125	6			25.5	1500	
Type 1 Retailer 6		130	6.5			26	1500	
Type 1 Retailer 7		90	7			27	1500	
Type 2 Retailer		70			8	12	2500	
Manufacturer	45	170	3.5	1.2	5	24	15,000	85,000
Supplier		550		2.2			15,000	150,000

Integral optimal solution to Example 1 in Table 3 shows that the minimal total cost 54,693.6 is obtained for 32 shipments of raw materials from supplier to manufacturer, 48 shipments of finished products from manufacturer to each retailer of Type-1 and 34 shipments of byproducts from manufacturer to Type-2 retailer. Integral optimal solutions to Example 2 are $M_1 = 5$, $M_2 = 5$, $M_3 = 15$ and $TC = 16{,}123.2$, which are tabulated in Table 4.

Table 3: Integral optimal solution to Example 1

	No. of shipments	Shipment size	Cost per year	Supply chain cost per year
Type 1 Retailer 1	48	104.167	1606.58	
Type 1 Retailer 2	48	156.25	1788.88	
Type 1 Retailer 3	48	208.333	1971.17	
Type 1 Retailer 4	48	312.5	2335.75	
Type 1 Retailer 5	48	364.583	2518.04	54,693.6
Type 1 Retailer 6	48	416.583	2700.33	
Type 1 Retailer 7	48	52.0833	1424.29	
Type 2 Retailer	34	264.706	1265.76	
Manufacturer	32	3459.82	37058.2	
Supplier			2024.61	

Table 4: Integral optimal solution to Example 2

	No. of shipments	Shipment size	Cost per year	Supply chain cost per year
Type 1 Retailer 1	5	150	710	
Type 1 Retailer 2	5	150	777.5	
Type 1 Retailer 3	5	150	835	
Type 1 Retailer 4	5	150	902.5	
Type 1 Retailer 5	5	150	955	16,123.2
Type 1 Retailer 6	5	150	1007.5	
Type 1 Retailer 7	5	150	975	
Type 2 Retailer	15	83.3333	833.333	
Manufacturer	5	1500	8832.34	
Supplier			295	

Conclusions

This study has presented a joint economic lot sizing coordinated inventory model for a three-tier supply chain involving a supplier, a manufacturer and multiple retailers. To make the environment green, the manufacturer uses byproducts on the demand of the consumers. Two types of retailers are considered in this model. Type-1 retailers' retail manufacturer's finished product, while Type-2 retailer trades manufacturer's byproducts. We have found an analytical solution to the model. Though the model is developed based on agricultural product, it is also useful in the supply chain of non-agricultural product. Attached algorithm is suitable for finding integral solution. Analysis of optimal solutions to numerical example problems shows that the low shipment cost leads to frequent shipments of smaller sizes. This study might be extended in different directions. Equal lot sizing policy may not be fruitful in some situations. Hence, equal and/or unequal lot sizing policy may be adopted for better results. Also, one can extend this work by considering warehouse capacity constraint or service level constraint or back ordering policy.

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