

Application of Linear Programming Approach for Determining Optimum Production Cost

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ABSTRACT

Cost optimization problem deals with that problem which aims to find out the most appropriate ways to fulfill the demand of a particular product of any manufacturing company with minimum cost. Linear programming is one of the most appropriate techniques for scheduling optimum cost of manufacturing. In this study, production schedule of a bicycle manufacturing company is taken into account. The mathematical formulation of the problem under consideration is performed by using linear programming approach. An operations research software, TORA (Temporary-Ordered Routing Algorithm), has been used in solving the problem and analyzing the results. Results reveal that a specific schedule has a great impact on optimizing the production cost.

Key words: Cost Optimization, Linear Programming, TORA, Manufacturing Company

INTRODUCTION

The main objective of a business is to earn profit. To ensure the attainment of this objective, an organization needs to ensure minimize its cost. Cost minimization not only ensures profit but also increases the market share of that company. To sustain in the marketplace for a long time, minimization of cost is a must. In a broader context, business organizations can be of two types. These are manufacturing (or product producing) organization and service providing organization. The scheduling of production has been a crucial issue for any manufacturing organization because an excess or insufficient supply of productions may lead to the loss of the company both in measures of time and money. Optimal planning and scheduling of productions will surely increase the productivity of the company.

With a view to minimizing the total production cost, a manufacturing organization should maintain proper scheduling of work for its labors. There is another cost associated with total cost. That is the inventory storage and handling cost. As a manufacturing organization produces physical products, it should keep the products in stock for maintaining proper flow of delivery and for fulfilling the need of future demand. So, a manufacturing

organization has to spend money in keeping the inventory in stock. Here, the organization should maintain an optimal level of inventory that ensures the least cost in one hand and the proper flow of inventory in another hand.

Sometimes, a manufacturing company may change the labor schedule due to financial and market demand related issues. The change may be increase in labor hours spent or decrease in labor hours spent. But, this should be kept at an acceptable limit. This means, the change (increase/decrease) should be congruent with the company policy. As a result, the rescheduling of labor mix needs to be made. But, the rescheduling of labor mix should be prepared in such a way that ensures minimum labor cost and minimum production cost. A linear programming (LP) technique may be effective to schedule the labor mix in the study that ensures the minimum labor cost.

To become successful in this complex business world, organizations should try to ensure excellence in all activities and also should try to capitalize on probable chances for achieving competitive advantage compared to others (Anieting et al., 2013). Veselovska (2014) said that manufacturing organizations recently have to face critical competitions which increase not only quality targets but also create problems on supply chain. Ibitoye et al. (2015)



investigated the effect of LP in the decision making process of entrepreneurs as an optimization tool for ensuring highest profit with the existence resources. Samples were taken from a fast food organization that faced few challenges for making meat pie, chicken pie and doughnut due to an upsurge in the price of raw materials. The results revealed that discontinuity needed to exist in the manufacturing of chicken pie and doughnut and that concentration should be made with the production of meat pie.

Felix et al. (2013) used farm activities to develop a LP model that showed options of selection that seemed achievable with a set of constant farm shortcomings and optimizing income while fulfilling other goals like the security of food. The results of traditional method were compared with the results that were achieved by using LP. Buresh-Oppenheim et al. (2011) and Baker (2011) examined the benefits and shortcomings of the usage of LP. Both of them considered the chance of applying these techniques for the long term manufacturing planning as the best crucial advantage. The comparative accuracy of these techniques was found as other advantage. Balogun et al. (2012) used LP method to get the highest amount of profit from the manufacturing of soft drink for Ilorin plant at Nigeria bottling company. They used LP of the operations of the company and they got optimum results using software based on simplex method.

In this study, data of a bicycle manufacturing company is considered to minimize the total cost of manufacturing. LP technique is used to present the problem mathematically. An operations research software TORA is used to solve and analyze the problem. The value of each decision variables is shown for the minimizing the total cost of manufacturing.

COMPONENTS OF A LP APPROACH

Like many other kinds of optimization approaches, LP is a mathematical model which has different components. The most important components of a LP model are:

- Defining key - decision variables
- Setting objective functions
- Writing mathematical expressions for constraints
- Non-negativity restriction
- Solving the mathematical model

Defining key - decision variables

Decision variables in a LP are the set of quantities that the decision makers would like to determine. They represent unknown decisions to be made in a LP. This is in contrast to problem data, which are values that are either given or can be simply calculated from what is given. The problem is solved when the best values of these variables have been identified. Basically, defining these variables of the problem is one of the hardest and/or most crucial steps in formulating a problem as a LP.

Setting objective functions

The objective function of a LP is an equation representing the profit or cost related to the decision variables. It is either to maximize profit or to minimize cost. It has to be linear in the decision variables, which means it must be the sum of constants multiplied by decision variables. It shows how each variable contributes to the value to be optimized in solving the problem. The coefficients in objective function indicate the contribution to the value of the decision variable of one unit of the corresponding variable.

Writing mathematical expressions for constraints

As a condition, LP problem must be operated within the limits of restrictions placed upon the problem, which the decision maker must always take into consideration (Tesfaye, 2016). Constraints are the limitations such as available resource capacity, daily working hours, raw material availability etc. This is done in the form of inequalities that describe mathematically the limitations on the resources.

Non-negativity restriction

In LP approaches, it is assumed that no variables will take on negative values, therefore each variable will have an associated constraint that limits it to values greater than or equal to zero.

Solving the mathematical model

A graphical method is the simplest method to solve a LP problem when the number of decision variables is two. However, most real-world LP problems have more than two decision variables and thus are too complex for graphical solution. Among the various methods of solving a LP problem the Simplex method is one of the most powerful methods (Andawei, 2014). Programming languages and commercial software are available to handle the solution of such complex problem.

PROBLEM SPECIFICATION

A bicycle company will be manufacturing both men's and women's models for its mountain bike JDC-400 bicycles during the next 2 months. Management wants to develop a production schedule indicating how many bicycles of each model should be produced in each month. Current demand forecasts call for 150 men's and 125 women's models to be shipped during the first month and 200 men's and 150 women's models to be shipped during the second month. Some additional data are presented in Table 1. In the last month, the company used a total of 1000 hours of labor. The company's labor relations policy will not allow the combined total hours of labor (manufacturing plus assembly) to increase or decrease by more than 100 hours from month to month. In addition, the company charges monthly inventory at the rate of 2% of the production cost based on the inventory levels at the

end of the month. The company would like to have at least 25 units of each model in inventory at the end of the 2 months. Management also wants to examine the change in production schedule if the monthly labors increase or decrease does not exceed 50 hours.

Table 1: Production cost, labor requirements and current inventory

Model	Production Costs	Labor Requirements (Hours)		Current Inventory
		Manufacturing	Assembly	
Men's	\$ 120	2.0	1.5	20
Women's	\$ 90	1.6	1.0	30

MODEL FORMULATION

The general form of a LP approach with n decision variables and m constraints can be expressed in the following form.

Optimize (Maximize or Minimize) $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to,

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq \text{or} = \text{or} \geq) b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq \text{or} = \text{or} \geq) b_2$$

$$\vdots \quad \quad \quad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq \text{or} = \text{or} \geq) b_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

The above model can also be expressed in a compact form as follows

Optimize (Maximize or Minimize) $Z = \sum_{j=1}^n c_j x_j$

Subject to,

$$\sum_{j=1}^n a_{ij} x_j (\leq \text{or} = \text{or} \geq) b_j, i = 1, 2, \dots, m$$

$$x_j \geq 0, j = 1, 2, \dots, n$$

Where $c_j, j = 1, 2, \dots, n$ represent the per unit cost (or profit) of decision variables $x_j, j = 1, 2, \dots, n$ to the value of the objective function. And $a_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n$ represent the amount of resources consumed per unit of the decision variables. The $b_i, i = 1, 2, \dots, m$ represent the total availability of the i^{th} resource. Z represents the measure of performance which can be either cost or profit.

Based on the data collected, there are 8 decision variables in our model which are assumed that

- x_1 = Amount of men's model in month 1
- x_2 = Amount of women's model in month 1
- x_3 = Amount of men's model in month 2
- x_4 = Amount of women's model in month 2

- x_5 = Inventory of men's model at the end of month 1
- x_6 = Inventory of women's model at the end of month 1
- x_7 = Inventory of men's model at the end of month 2
- x_8 = Inventory of women's model at the end of month 2

Now the formulated model is given as

Minimize $Z = 120x_1 + 90x_2 + 120x_3 + 90x_4 + 2.4x_5 + 1.8x_6 + 2.4x_7 + 1.8x_8$

Subject to,

Demand satisfaction : $x_1 - x_5 \geq 130$

$$x_2 - x_6 \geq 95$$

$$x_3 + x_5 - x_7 \geq 200$$

$$x_4 + x_6 - x_8 \geq 150$$

Ending inventory requirement : $x_7 \geq 25$

$$x_8 \geq 25$$

Laborsmoothing: $3.5x_1 + 2.6x_2 \geq 900$

$$3.5x_1 + 2.6x_2 \leq 1100$$

$$3.5x_1 + 2.6x_2 - 3.5x_3 - 2.6x_4 \leq 100$$

$$-3.5x_1 - 2.6x_2 + 3.5x_3 + 2.6x_4 \leq 100$$

Non - negativity restriction : $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$

Now, introducing the slack and surplus variables to convert the above LP model to its standard form, it gives:

Minimize $Z = 120x_1 + 90x_2 + 120x_3 + 90x_4 + 2.4x_5 + 1.8x_6 + 2.4x_7 + 1.8x_8$

Subject to,

$$x_1 - x_5 - s_1 = 130$$

$$x_2 - x_6 - s_2 = 95$$

$$x_3 + x_5 - x_7 - s_3 = 200$$

$$x_4 + x_6 - x_8 - s_4 = 150$$

$$x_7 - s_5 = 25$$

$$x_8 - s_6 = 25$$

$$3.5x_1 + 2.6x_2 - s_7 = 900$$

$$3.5x_1 + 2.6x_2 + s_8 = 1100$$

$$3.5x_1 + 2.6x_2 - 3.5x_3 - 2.6x_4 + s_9 = 100$$

$$-3.5x_1 - 2.6x_2 + 3.5x_3 + 2.6x_4 + s_{10} = 100$$

$$x_1, x_2, \dots, x_8, s_1, s_2, \dots, s_{10} \geq 0$$

In the following section, the above LP problem is solved by using the computer software TORA.

RESULTS AND DISCUSSION

The above model was solved by using the software TORA. Table 2 shows the values of eight decision variables and contribution of these variables in the objective function. The minimum cost of the company after satisfying their all requirements is \$ 67156.03. The company should produce 63 men's model and no women's model as inventory in the first month so that the inventory at the end of second month satisfies. Table 3 shows the values of slack or surplus variables associated with each constraint. Also the dual price of each constraint is shown in Table 3.

In the case of less than or equal (\leq) constraint, the dual price gives the value of having one more unit of the resource represented by that constraint. On the other hand, in the case of greater than or equal (\geq) constraint the dual price gives the cost of meeting the last unit of the minimum production target. The company would use 922.25 and 1022.25 hours of labor in the first and second month respectively where they used 1000 hours of labor at the previous month.

Table 2: Contribution of decision variables in objective function

Variables	Value	Objective coefficient	Objective value contribution
x_1	192.93	120.00	23151.43
x_2	95.00	90.00	8550.00
x_3	162.07	120.00	19448.57
x_4	175.00	90.00	15750.00
x_5	62.93	2.40	151.03
x_6	0.00	1.80	0.00
x_7	25.00	2.40	60.00
x_8	25.00	1.80	45.00

Table 3: Values of slack or surplus variables and dual price of constraints

Constraints	RHS	Slack -/Surplus +	Dual Price
>	130.00	0.00	118.80
>	95.00	0.00	89.11
>	200.00	0.00	121.20
>	150.00	0.00	90.89
>	25.00	0.00	123.60
>	25.00	0.00	92.69
>	900.00	22.25+	0.00
<	1100.00	177.75-	0.00
<	100.00	200.00-	0.00
<	100.00	0.00	0.34

To accommodate the new policy, the right-hand sides of the four labor-smoothing constraints must be changed to 950, 1050, 50, 50 respectively. Then the new total cost will be \$ 67175.06 which is more \$ 19.03 than the cost of existing policy. So, the total production cost will increase if the company changes the monthly labor policy by increasing or decreasing more than by 100 hours by increasing or decreasing more than by 50 hours.

CONCLUSION

The data of a bicycle manufacturing company were analyzed to make a schedule of production for next two months. The mathematical formulation was carried out by LP technique and the problem was solved by TORA. A production schedule was made for next two months where the total cost would be \$ 67156.03. Our investigation also showed that the existing labor policy of the company was the best than the new labor policy.

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